zero pressure gradient; (iii) the omission of any specification of surface roughness of plate. Moreover, there is a question regarding the validity of the expression for Stanton number, equation (11), and the abscissa variable in Fig. 4.

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A COMMENT ON TURBULENT MOMENTUM DIFFUSIVITY WITHIN A CIRCULAR TUBE*

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WHILE Hooper's [1] determination of the mixing length and eddy viscosity from a simple model of the eddy structure is interesting, further discussion of the points marked experimental in Figs. 2 and 3 is needed. It will be shown that the calculation of these values from the experimental velocity distribution involves a supplementary assumption, and that the two sets of values are based on inconsistent assumptions.

The expressions for the mixing length and eddy viscosity can be written as:

$$\frac{l_m}{R} = \left(\frac{r}{R}\right)^{1/2} \div \frac{\mathrm{d}W_d}{\mathrm{d}\,(r/R)} \tag{1}$$

and

$$\frac{\epsilon}{W_\tau R} = \frac{r}{R} \div \frac{\mathrm{d}W_d}{\mathrm{d}\,(r/R)}. \tag{2}$$

The symbols are defined in [1], except for W_d, the velocity defect which is $(W_{\text{center}} - W) \div W_\tau$.

In order to determine the "experimental" points, it is necessary to differentiate the velocity distribution measured at discrete points. Any numerical differentiation procedure depends on an assumption about the form of the functional relation involved and the values of the derivative can be quite sensitive to the assumption. In the present case this is particularly important because both (1) and (2) are indeterminate as $r/R \to 0$.

It is easily seen that the mixing length only has a finite value on the pipe axis if $W \sim (r/R)^{3/2}$ for small values. If the exponent is less than 3/2, the mixing length approaches zero on the axis, if it is greater the mixing length becomes infinite. This was pointed out by Prandtl [2] in 1925, and undoubtedly used by Nikuradze in evaluating the derivative.

On the other hand, the eddy viscosity is finite on the axis only for $W_d \sim (r/R)^2$. Therefore, the finite values on the axis in Figs. 2 and 3 are inherently inconsistent.

In Table 1, the values of W, r, $W_c - W/r^{3/2}$, and $W_c - W/r^2$ are tabulated for a typical Nikuradze [3] traverse and for one reported by Stanton [4]. The indeterminacy close to the axis is clearly shown, and it would appear that the r^2 assumption (i.e. finite eddy. viscosity) is more plausible than the $r^{3/2}$ assumption Furthermore, as the tabulation indicates, the distribution is parabolic over most of the pipe it is very difficult to see how Schlichting [5] obtained the variation shown in Fig. 3. The original reference gives no details.

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Table 1

(a) Nikuradze Traverse 9·94 cm pipe, $Re = 106 \times 10^3$				(b) Stanton Traverse 4·93 cm pipe			
W (cm/s)	r (cm)	$\frac{W_c - W}{r^{3/2}}$	$\frac{W_c - W}{r^2}$	W (cm/s)	r (cm)	$\frac{W_c - W}{r^{3/2}}$	$\frac{W_c - W}{r^2}$
141·4	0·0			1525	0		
141·3	0·10	$\overline{3\cdot1}$	$\overline{10}$	1505	0·45	$\overline{66}$	$\overline{0\cdot99}$
141·2	0·20	2·3	5	1475	0·70	86	102
140·8	0·50	1·7	2·4	1440	0·96	90	92
139·5	0·99	1·9	1·9	1395	1·21	98	89
137·6	1·49	2·1	1·7	1346	1·47	100	83
135·0	1·99	2·3	1·6	1278	1·72	110	83
132·0	2·49	2·4	1·5	1202	1·97	117	83
128·0	2·98	2·6	1·5	1149	2·10	124	85
123·0	3·48	2·8	1·5	1084	2·23	133	89
116·5	3·98	3·1	1·6	1049	3·28	138	92
112·0	4·22	3·4	1·5	1025	2·32	141	93
106·2	4·47	3·7	1·8				

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